Horizon Dynamics of Evaporating Black Holes in a Higher Dimensional Inflationary Universe

Manasse R. Mbonye

Physics Department, University of Michigan, Ann Arbor, Michigan 48109

Abstract

Spherically symmetric Black Holes of the Vaidya type are examined in an asyptotically de Sitter, higher dimensional spacetime. The various horizons are identified and located. The structure and dynamics of such horizons are studied.

PACS number(s): 04.70.Nr, 04.50.+h, 04.70.Bw, 04.40.Dy

I. INTRODUCTION

The notion that our universe may have begun from a hot big-bang forms the pillar of the standard cosmological model, a model that plots the time evolution of the universe. This model holds that soon after the big-bang ($\sim 10^{-36}s$) the temperature of the universe drops below the critical grand-unified theory temperature ($T_{GUT} \sim 10^{14} GeV$). In this over-cool sub-stable state, the universe is dominated by vacuum energy-density with an effective cosmological constant

$$\Lambda = \frac{2}{15}\pi^3 \left(\frac{T_{GUT}^2}{m_p}\right)^2$$

where m_p is the Planck mass ($\sim 10^{19}\, GeV$). This energy then drives the same universe into a de Sitter-like exponential inflation [1]. The standard cosmological model has been very successful in explaining the origins of most of the properties of our universe: from the very small to the very big, from nucleosynthesis to large scale structure formation and the expansion of the universe.

This standard model of cosmology has not, however, been able to address the physics of the events prior to the inflationary era. Understanding this very short but very important era of the universe requires a theory that incorporates General Relativity and Quantum Theory. Such an effort translates into unifying gravity with the other three forces of nature. Judging

by what has been learned from such candidates as Supergravity [2] and now Superstrings [3], it appears that such a unifying theory will likely require the universe to be multi-dimensional. In Superstrings, for example, the view is that the universe started out in a higher-dimensional phase. Four of the dimensions then expand leaving behind the rest in the regime of the planck length. In the field theoretic-limit of Superstring theories, gravity is described reasonably accurately by multi-dimensional Einstein field equations [3]. In this unification scheme, internal symmetries can be traced to the spacetime associated with the extra dimensions [4]. Gauge invariance then assumes the same status as spacetime invariance and internal quantum numbers such as electric charge, believed to result from symmetry motions in the extra dimensions, are now brought to the same footing as energy and momentum.

In searching for possible effects of the extra dimensions it makes sense, therefore, to evolve the Friedman-Robertson-Walker model of our universe back in time to the early de Sitter phase where one reaches energies at which such extra dimensions may be resolvable (see [4] and Refs. therein). Coincidentally, this too turns out to be the era when primordial black holes may have been produced $[5-9]$. For this and other reasons (like the distinctly high curvature nature of the spacetime around them) black holes will continue to be important probes in any quantum theory of gravity, and in the quest to understand the role of multi-dimensionality in the early (and possibly present? (see [10])) evolution of our universe. The recent findings of links between black hole entropy and topological structures of the extra dimensions (see for example [11] and [12]) not only puts high-dimensionality to a stronger footing but also emphasizes the role black holes will play as probes in such future research. Naturally this calls for an understanding of the dynamics of black holes in higher dimensions, particularly under conditions that mimic those of the early universe. In this paper we begin a study of the dynamics of such a black hole in such a setting.

Several solutions to the Einstein equations of localized sources in higher dimensions have been obtained in the recent years. This includes the higher dimensional generalizations of the Schwarzschild and the Reisner-Nordstrom solutions [13, 14], the Kerr solution [15, 16]

and the Vaidya solution [17]. Recently the metric of a radiating black hole in a de Sitter background, that is a generalization of the Mallett [18] metric, has been written down [19, 20]. In the present work our aim is to demonstrate that the dynamics of a radiating black hole in a higher dimensional cosmological background can be sensibly discussed. First, we seek to identify and locate the various horizons. After this we then go on to study the structures and discuss the dynamics of such horizons. It is shown, at each stage, that all the results we obtain reduce to the well known Mallett [21] results as we go down to four dimensions, and to make this transparent our analysis is closely modelled to that of Mallett.

In Section II we introduce the working metric and the theoretical background. In Section III we derive equations for the horizons. We solve these equations and use the solutions to identify and locate the various horizons in the problem. In Section IV we take up the issue of the structure of such horizons and study their dynamics. In Section V we conclude the discussion.

II. THE METRIC AND THE THEORY

A. The Metric

In this treatment we wish to consider a radiating black hole introduced in an N dimensional de Sitter space-time. We suppose, for simplicity, that such a black hole is reasonably modelled by an imploding shell of negative-energy-density null fluid in the de Sitter universe. In advanced time, comoving, coordinates the metric [19, 20] is given by the line element ,

$$ds^2 = -\left[1 - \frac{2G_N m(v)}{nr^n} - \frac{2\Lambda}{(n+1)(n+2)}r^2\right]dv^2 + 2dvdr + d\Omega_{n+1}^2 \tag{2.1}$$

where $n = N - 3$, $m(v)$, the mass, is a monotonically decreasing function of the advanced time coordinate v, G_N is the N-dimensional gravitational constant, Λ is the cosmological constant and

$$d\Omega_{n+1}^2 = r^2\left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + ... + \sin^2\theta_1 \sin^2\theta_2 .... \sin^2\theta_n d\theta_{n+1}^2\right) \tag{2.2}$$

is the line element on the $(n+1)$-sphere. For N = 4 the geometry reduces to that of the Vaidya-Mallett space-time [18].

In the absence of the cosmological background, Λ, the usual luminosity, L_0, of the black hole is defined [19] from the only non-vanishing component of the energy-momentum tensor,

$$T_v^r = T_{vv} = \frac{(n+1)}{8\pi n}\frac{\dot{m}(v)}{r^{n+1}} \tag{2.3}$$

as

$$L_0 = -\dot{m} \tag{2.4}$$

Here and henceforth $\dot{m}(v)$ and $m'(v)$ denote, as usual, derivatives with respect to the time and space coordinates, respectively. The Luminosity which is bounded from above, $L_0 < 1$, is measured in regions where $\frac{d}{dv}$ is time-like.

One can introduce a basis of vectors at every point in this spacetime. Two such vectors β_a and l_a span the radial-temporal subspace and are given by

$$\beta_a = \delta_a^v \tag{2.5}$$

and

$$l_a = -\frac{1}{2}\left[1 - \frac{2G_N m(v)}{nr^n} - \frac{2\Lambda}{(n+1)(n+2)}r^2\right]\delta_a^v + \delta_a^r \tag{2.6}$$

while the rest of the $(N-2)$ vectors are defined on the $n+1$-sphere and induce on the latter a tensor field γ_{ab} of the form

$$\gamma_{ab} = r^2\left(\delta_a^{\theta_1}\delta_b^{\theta_1} + \sin^2\theta_1\delta_a^{\theta_2}\delta_b^{\theta_2} + ... + \sin^2\theta_1\sin^2\theta_2....\sin^2\theta_n\delta_a^{\theta_{n+1}}\delta_b^{\theta_{n+1}}\right). \tag{2.7}$$

The vectors satisfy the conditions

$$\beta_a\beta^a = l_a l^a = 0,\ \gamma_{ab}\beta^b = \gamma_{ab}l^b = 0,\ \beta_a l^a = -1. \tag{2.8}$$

One can do a null-vector decomposition of the above metric in this basis so that

$$g_{ab} = -\beta_a l_b - l_a\beta_b + \gamma_{ab}. \tag{2.9}$$

B. Deformation of relativistic Membranes

The structure and dynamics of horizons of such non-static metrics can be approached from the non-perturbative description of deformation of relativistic membranes. In general, one considers the evolution of such deformations of an arbitrary D-dimensional membrane in an arbitrary N-dimension space-time. A significant amount of literature has been written on the subject of such deformations [22, 23, 24]. The quantities that characterize how a variation in the symmetry of a membrane evolves are the expansion rate, θ, the shear rate, σ, and the vorticity (twist), ω. For the general D-dimensional membrane case, the equations for the deformations have usually yielded no simple clear interpretation of these quantities [23]. Recently, Zafiris [24] has made some progress in addressing the problem. However, for the special $D = 1$ case in an N dimensional background one can still generalize the Carter [22] form of the Rachaudhuri [25] equation so that

$$\frac{d\theta}{dv} = \kappa\theta - \left(\gamma_c^c\right)^{-1}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{ab}l^a l^b \tag{2.10}$$

with θ, σ, and ω taking their usual physical meaning. Here R_{ab} is the N-dimensional Ricci tensor, γ_c^c is the trace of the projection tensor for null geodesics and κ is to be identified as the surface gravity. The latter is given by

$$\kappa = -\beta^a l^b \nabla_b l_a, \tag{2.11}$$

where ∇ is the covariant derivative operator. For infinitesimally neighboring members of the congruence separated by a relative separation vector $d\mathbf{x}$ the rate of the change of separation is given [22] by

$$\frac{1}{2}(\dot{ds^2}) = \theta_{ab}dx^a dx^b. \tag{2.12}$$

The expansion rate θ, of a null geodesic congruence is then given by the trace of the expansion tensor as

$$\theta = \theta_a^a = \gamma^{ab}\nabla_a l_b. \tag{2.13}$$

It follows then that

$$\nabla_a l^a = \frac{\partial l_a}{\partial x^a} + \Gamma^a_{ac} l^c = \kappa + \theta. \tag{2.14}$$

And clearly in flat space-time θ vanishes since the connection coefficients Γ^a_{ac} will.

III. LOCATION OF THE HORIZONS

Spherically symmetric irrotational space-times, such as under consideration, are vorticity and the shear free. The structure and dynamics of the horizons are then only dependent on the expansion, θ. Following York [26] we note that to $O(L_0)$ the evolution of an apparent horizons (AH) is to satisfy the requirement that $\theta \simeq 0$, while that of an event horizons (EH) is to satisfy the requirement that $\frac{d\theta}{dv} \simeq 0$.

A. The Apparent Horizons

We have written the general expression for the expansion θ as

$$\theta = \gamma^{ab} \nabla_a l_b \tag{3.1}$$

Using equations (2.1), (2.6) and (2.7) in (3.1) yields

$$\theta(r) = \frac{n+1}{2r} \left[1 - \frac{2G_N m(v)}{nr^n} - \frac{2\Lambda}{(n+1)(n+2)} r^2 \right]. \tag{3.2}$$

Consider now the function $f(r) = -\frac{2}{n+1} r\theta$. Since the York conditions require that at the (AHs) θ and hence f vanish, it follows from equation (3.2) that these surfaces will satisfy

$$r^{n+2} - \frac{(n+1)(n+2)}{2\Lambda} r^n + \frac{(n+1)(n+2)}{n} \frac{G_N m(v)}{\Lambda} = 0. \tag{3.3}$$

As it stands equation (3.3) will obviously not admit simple closed form solutions. It is possible, however, to put this equation in a useful form that yields solutions which for practical purposes can, reasonably and justifiably, be taken as the working solutions to the problem at hand. To this end it is helpful to first gain some insight in the nature of the

function $f(r)$ whose roots satisfy (3.3). One notices that the turning points for the function $f(r)$ are located at points where

$$\left[r^2 - \frac{n(n+1)}{2\Lambda}\right] r^{n-1} = 0. \tag{3.4}$$

The derivative $\frac{df}{dr}$ then vanishes at points $r = 0$ and $r = \pm\left[\frac{n(n+1)}{2\Lambda}\right]^{\frac{1}{2}}$. From this one then notices that between $r = +\infty$ and $r = +\left[\frac{n(n+1)}{2\Lambda}\right]^{\frac{1}{2}}$ the function changes sign from positive to negative which implies that the function has a root imbedded in between these two values. One notices further, that between $r = +\left[\frac{n(n+1)}{2\Lambda}\right]^{\frac{1}{2}}$ and $r = 0$, $f(r)$ does again change signs this time from negative to positive, and this betrays the existence of yet a second root. We then establish that the function $f(r)$ has at least two real roots. More importantly we establish that for $r > 0$ this function has two, and only two, real roots. This is an encouraging result consistent with our expectations [21] that there should be two apparent horizons r_{AH}^{-} and r_{AH}^{+} associated with the black hole and the cosmological constant, respectively.

The case of the r^{n-1} turning points at $r = 0$ suggests the existence of other roots for the function $f(r)$. Such roots are actually complex and would suggest the existence of *ghost horizons* (at least for an observer in our four dimensional spacetime). I will defer, for now, further speculation of their significance and other issues about the turning points for a future discussion. It is, however, interesting to note that such *ghost horizons* only start to appear at five dimensions $(n = 2)$ and persist for higher dimensions.

To put equation (3.3) in a more manageable form, it is useful to institute a change of variables. Thus setting

$$r = k\xi, \tag{3.5}$$

where $k = \sqrt{\frac{(n+1)(n+2)}{2\Lambda}}$ equation (3.3) can be cast in a form

$$\xi^{n+2} - \xi^{n} + \beta(n) = 0, \tag{3.6}$$

where

$$\beta(n) = \frac{2G_N m(v)}{n}\left[\frac{2\Lambda}{(n+1)(n+2)}\right]^{\frac{n}{2}}. \tag{3.7}$$

One notes that since in our model $\Lambda > 0$, and so $\beta(n) > 0$, then (3.6), along with the necessary positive reality of r, imply that $0 < \xi < 1$ and $0 < \beta < 1$.

Now set $\xi = 1 - x$, where $0 < x < 1$ to cast equation (3.6) in the form

$$x(2-x)(1-x)^n - \beta(n) = 0. \tag{3.8}$$

The above limits imposed on ξ and hence x suggest one can seek approximate solutions through expansion of the quantity $(1-x)^n$ in equation (3.8). Thus keeping in the equation terms up to x^4 gives

$$x^4 + ax^3 + bx^2 + cx + d = 0, \tag{3.9}$$

where

$$a = -\frac{6n}{(n-1)(2n-1)}, \; b = \frac{6(2n+1)}{n(n-1)(2n-1)}, \; c = -\frac{12}{n(n-1)(2n-1)},$$

and

$$d = \frac{6}{n(n-1)(2n-1)}\beta(n). \tag{3.10}$$

We should mention that this expansion is strictly valid for $n > 2$. The solutions for the 5-dimensional ($n = 2$) case are actually exact. Further, the solutions have the right limits when $n = 1$ and for this case one does recover the exact Mallett's cubic equation [21] for the 4-dimensional case from either of the equations (3.3) or 3.6). In general this approximation is good in the limit $x \to 0$, $\xi \to 1$. It, however, breaks down when $x \to 1$, $\xi \to 0$. In this latter limit the solution to (3.6) simply goes to $\xi = [\beta(n)]^{\frac{1}{n}} \Longrightarrow r = \left(\frac{2Gm}{n}\right)^{\frac{1}{n}}$ and the geometry loses knowledge of the cosmological constant.

Equation (3.9) has a resolvent cubic equation that can be that can be written in the form

$$y^3 + py + q = 0 \tag{3.11}$$

where $p = \frac{1}{3}\left[\left(3ac - b^2\right) - 12d\right]$ and $q = \frac{1}{27}\left[9abc - 2b^3 - 27c^2 - 9\left(3a^2 + b\right)d\right]$. Equation (3.11) admits three solutions. One such solution that is real for the parameters p, q as defined above can be written as

$$y_1 = 2\sqrt{\frac{-p}{3}}\cos\frac{1}{3}\varphi, \tag{3.12}$$

where $\frac{\pi}{2} < \varphi < \pi$ is given by $\varphi = \arccos\left(\frac{3q}{2\sqrt{\frac{-p^3}{3}}}\right)$.

There are four solutions to the quartic equation (3.9) in x. The only physically interesting solutions to equation (3.9) consistent with $r = k\xi = k\left(1 - x\right)$ should be real and satisfy $x < 1$. There are two such solutions obtained by letting

$$R = \sqrt{\frac{a^2}{4} - b + \left(2\sqrt{\frac{-p}{3}}\right)\cos\frac{1}{3}\varphi}$$

and

$$D = \sqrt{\frac{3a^2}{4} - R^2\left(p,\varphi\right) - 2b + \frac{4ab - 8c - a^3}{4R\left(p,\varphi\right)}} \tag{3.13}$$

These are:

$$x_\pm = -\frac{1}{4}a + \frac{1}{2}R\left(p,\varphi\right) \pm \frac{1}{2}D\left(p,\varphi\right). \tag{3.14}$$

Note that all the time dependency of R and so of D is expressed in p and φ via $d\left(\beta\left(m\left(v\right)\right)\right)$.

Using the solutions in equation (3.14) and recalling that $\xi = 1 - x$ we obtain two values ξ_1 and ξ_2. On applying these results to equation (3.5) i.e. $r = k\xi = k\left(1 - x\right)$ we find two solutions $r_1 = r^-_{AH}\left(v\right)$ and $r_2 = r^+_{AH}\left(v\right)$ such that

$$r^-_{AH}\left(v\right) = r_1 \simeq k\left\{1 - \frac{1}{4}\left[2D\left(p,\varphi\right) + 2R\left(p,\varphi\right) - a\right]\right\}, \tag{3.15}$$

and

$$r^+_{AH}\left(v\right) = r_2 \simeq k\left\{1 + \frac{1}{4}\left[2D\left(p,\varphi\right) - 2R\left(p,\varphi\right) + a\right]\right\} \tag{3.16}$$

In the limit $n \to 1$ one recovers the well known solutions [21]. Thus

$$\lim_{n=1} r^{-}_{AH}(v) = -\left(\frac{2}{\sqrt{\Lambda}}\right)\cos\left(\frac{1}{3}\Psi + \frac{1}{3}\pi\right) \tag{3.17}$$

and

$$\lim_{n=1} r^{+}_{AH}(v) = \left(\frac{2}{\sqrt{\Lambda}}\right)\cos\left(\frac{1}{3}\Psi\right) \tag{3.18}$$

where now $\left(\frac{\pi}{2} < \Psi(v) < \pi\right) = \arccos\left[-3m(v)\sqrt{\Lambda}\right]$. (Note, however, that φ is not simply related to Ψ by taking the limit $n \to 1$). Further, in the limit $\Lambda \longrightarrow 0$ and $n \to 1$ we recover the black hole apparent horizon $r^{-}_{AH}(v) \to 2m(v)$ and in the limit $m \to 0$ and $n \to 1$ we find, as one expects that $r^{+}_{AH}(v) \to \sqrt{\frac{\Lambda}{3}}$. Hence our solutions reduce to all the well known solutions [21] in a four dimensional spacetime. Consequently, we identify the locus of $r^{-}_{AH}(v)$ in equation (3.15) as the black hole apparent horizon (AH^{-}) while we identify the locus of $r^{+}_{AH}(v)$ in equation (3.16) as the de Sitter apparent horizon (AH^{+}), for a black hole in an N-dimensional background with a cosmological constant.

B. The Event Horizons

The event horizons are null surfaces. To $O(L_0)$ the evolution of these surfaces can be determined from the second of the York conditions that $\frac{d\theta}{dv} \simeq 0$. We first show that for a black hole imbedded in an N-dimensional de Sitter background this condition is satisfied. We shall then solve the resulting equations to locate the event horizons and study their structure.

The surface gravity κ in this spacetime is given from (2.11) by

$$\kappa = \frac{G_N m(v)}{r^{n+1}} - \frac{2\Lambda}{(n+1)(n+2)} r. \tag{3.17}$$

Consider now the acceleration of the geodesics $\frac{d^2 r}{dv^2}$ in our space-time, parametrized by v.

Since from the line element (2.1) we have that

$$\frac{dr}{dv} = \frac{1}{2}\left[1 - \frac{2G_N m(v)}{n r^n} - \frac{2\Lambda}{(n+1)(n+2)} r^2\right] \tag{3.18}$$

then

$$\frac{d^2r}{dv^2}=\frac{G_N L_0}{nr^n}+\kappa\frac{dr}{dv} \tag{3.19}$$

Equations (3.2) and (3.18) in (3.19) give

$$\frac{d^2r}{dv^2}=\frac{G_N L_0}{nr^n}+\kappa\theta\frac{r}{n+1}. \tag{3.20}$$

But the event horizon is a null surface and satisfies the general requirement that null-geodesic congruencies have a vanishing acceleration. Thus at the event horizon (3.20) takes the form

$$\kappa\theta_{EH}+\frac{n+1}{n}\frac{G_N L_0}{(r_{EH})^{n+1}}=0. \tag{3.21}$$

Now the Einstein field equations for the $N(N=n+3)$-dimensional space-time are [4]

$$R_{ab}=8\pi G_N\left[T_{ab}-\frac{1}{n+1}g_{ab}T_c^c\right]+\frac{2\Lambda}{n+1}g_{ab}. \tag{3.22}$$

Using equations (2.3), (2.6), and (2.8) with (3.22) one finds that

$$R_{ab}l^a l^b=\frac{(n+1)}{n}\frac{G_N}{r^{n+1}}\dot{m}(v) \tag{3.23}$$

For a spherically symmetric irrotational space-time, such as under consideration, the vorticity ω and the shear σ vanish and the Raychauduri equation (2.10) reduces to

$$\frac{d\theta}{dv}=\kappa\theta-R_{ab}l^a l^b-(\gamma_c^c)^{-1}\theta^2. \tag{3.24}$$

Equations (3.21) and (3.23) when substituted in (3.24) show (on neglecting the term in θ^2) that, indeed, the York condition for the event horizon is satisfied and we have

$$\left(\frac{d\theta}{dv}\right)_{EH}\simeq 0. \tag{3.25}$$

The event horizons (EHs) in our problem are therefore located by equation (3.25). Using equation (3.2) equation (3.25) can be written in the form

$$r^{n+2}-\frac{(n+1)(n+2)}{2\Lambda}r^n+\frac{(n+1)(n+2)}{n}\frac{G_N m^*(v)}{\Lambda}=0. \tag{3.26}$$

where m^* is some effective mass given by

$$m^*(v) = m(v) - \frac{L_0}{\kappa} \tag{3.27}$$

Equation (3.26) for the location of event horizons is exactly of the same form as its counterpart equation (3.3) for location of the apparent horizons with the mass m replaced by the effective mass m^* as defined in equation (3.27). Hence borrowing from our previous techniques in solving equation (3.3) we can immediately write down the solutions to 3.26 as

$$r_{EH}^-(v) \simeq k\left\{1 - \frac{1}{4}\left[2D^*(p,\varphi) + 2R^*(p,\varphi) - a^*\right]\right\}, \tag{3.28}$$

and

$$r_{EH}^+(v) \simeq k\left\{1 + \frac{1}{4}\left[2D^*(p,\varphi) - 2R^*(p,\varphi) + a^*\right]\right\}, \tag{3.29}$$

where $*$ means $m(v) \to m^*(v) = m(v) - \frac{L_0}{\kappa}$ and $\frac{1}{2}\pi < \varphi^* < \pi$. In the limit $n \to 1$ equation (3.28) reduces to,

$$\lim_{n=1} r_{EH}^-(v) = -\left(\frac{2}{\sqrt{\Lambda}}\right)\cos\left(\frac{1}{3}\Psi^* + \frac{1}{3}\pi\right) \tag{3.30}$$

while

$$\lim_{n=1} r_{EH}^+(v) = \left(\frac{2}{\sqrt{\Lambda}}\right)\cos\left(\frac{1}{3}\Psi^*\right) \tag{3.31}$$

where $\left(\frac{\pi}{2} < \Psi^*(v) < \pi\right) = \arccos\left[-3m^*(v)\sqrt{\Lambda}\right]$. These limiting cases then reproduce exactly the known equations [21] for locations of the event horizons in such a four dimensional spacetime. Further, as one switches Λ off one finds from equations (3.17) and (3.3) that the surface gravity of the black hole measured at the apparent horizon becomes

$$\kappa = \frac{n}{2}\left[\frac{2G_N m(v)}{n}\right]^{-\frac{1}{n}} \tag{3.32}$$

Equation (3.26) along with equation (3.32) imply that in the limit $\Lambda \to 0$, then

$$r_{EH}^-(v) \to \left(\frac{2G_N}{n}\right)^{\frac{1}{n}}\left[m(v) - \frac{2}{n}\left(\frac{2G_N m(v)}{n}\right)^{\frac{1}{n}} L_0\right] \tag{3.33}$$

Equation (3.33) locates the event horizon of a radiating black hole imbedded in a higher dimensional Schwarzschild spacetime. And as the dimensionality is reduced to four, it is clear that

$$r_{EH}^{-}(v) \rightarrow 2Gm(v)(1-4GL_0). \tag{3.34}$$

This is the exact result originally obtained by York [26] and later verified by Mallett [18]. It follows then that in equation (3.28) $r_{EH}^{-}(v)$ does indeed represent the locus of the black hole event horizon in a higher dimensional spacetime with a cosmological constant. Further, the $r_{EH}^{+}(v)$ in equation (3.29) is seen to give the locus of the cosmological event horizon $r_{EH}^{+}(v)$.

For an observer positioned at $r_{AH}^{-}(v) < r < r_{AH}^{+}(v)$, the region $EH^{-} \cap AH^{-}$ represents the quantum ergosphere [26]. The ordering of the horizons can now be made. One finds from our results that $EH^{-} < AH^{-} < AH^{+} < EH^{+}$. The finite location of the cosmological event horizon EH^{+} is, as can be inferred from equation (3.29), due to the presence of the cosmological constant. One finds, indeed, that in the event $\Lambda \rightarrow 0$ then $EH^{+} \rightarrow \infty$. Again all these results are consistent with the known results for the $N = 4$ case.

IV. STRUCTURE AND DYNAMICS OF THE HORIZONS

We now turn to the problem of the structure and motion of the various horizons obtained in our results above.

A. Structure of the Apparent Horizons

In the foregoing discussion we have found the locations of the apparent horizons. We now deduce the structure of these surfaces. Since at the apparent horizons the expansion θ vanishes then from equations (3.3) and (2.1) the metric on such surfaces will take the form $ds^2 = 2dvdr + d\Omega_{n+1}^2$. One finds that equations (2.1), and (3.15) as defined by equations (3.13) and (3.14) will induce on the surface r_{AH}^{-} a metric of the form

$$ds^2\mid_{r=r_{AH}^-}=\frac{k}{2}\alpha_-\left(p,\varphi\right)\left[\rho\frac{\sin\frac{1}{3}\varphi}{\sin\varphi}+\sigma\cos\frac{1}{3}\varphi\right]\frac{dm}{dv}dv^2+d\Omega_{n+1}^2, \tag{4.1}$$

where

$$\begin{aligned}\alpha_-\left(p,\varphi\right) &= \left(DR\right)^{-1}\left[2R+\frac{\left(4ac-8c-a^3\right)}{4R}-1\right],\ \rho=\frac{1}{3}\sqrt{\frac{-p}{3}}\frac{d}{dm}\left(\frac{3q}{2\sqrt{\frac{-p^3}{3}}}\right),\\ \sigma &= \frac{2d}{m\left(v\right)\sqrt{-3p}}.\end{aligned} \tag{4.2}$$

Noting that α,ρ,σ are all positive quantities then for $\frac{\pi}{2}<\varphi<\pi$, $\frac{dm}{dv}<0$ contributes the only negative quantity in equation (4.2). We conclude, on this basis, that according to equation (4.1) the apparent horizon surface $r_{AH}^-\left(v\right)$ of an evaporating black hole in a higher dimensional de Sitter spacetime is timelike.

Similarly, for $\frac{\pi}{2}<\varphi<\pi$ equations (2.1), and (3.16) as defined by equations (3.13) and (3.14) will induce on the surface r_{AH}^+ a metric of the form

$$ds^2\mid_{r=r_{AH}^+}=-\frac{k}{2}\alpha_+\left(p,\varphi\right)\left[\rho\frac{\sin\frac{1}{3}\varphi}{\sin\varphi}+\sigma\cos\frac{1}{3}\varphi\right]\frac{dm}{dv}dv^2+d\Omega_{n+1}^2, \tag{4.3}$$

where ρ and σ are as given in equation (4.2) and

$$\alpha_+\left(p,\varphi\right)=\left(DR\right)^{-1}\left[2R+\frac{\left(4ac-8c-a^3\right)}{4R}+1\right] \tag{4.4}$$

It follows, then from use of equations (4.2) and (4.4) that the cosmological apparent horizon r_{AH}^+ in a higher dimensional spacetime (as located by equation (4.3)) is spacelike.

In the limit $n\rightarrow 1$, one finds that

$$ds^2\mid_{r=r_{AH}^-}=-\frac{4\sin\left(\frac{1}{3}\Psi+\frac{1}{3}\pi\right)}{\sin\Psi}\frac{dm}{dv}dv^2+d\Omega_{n+1}^2, \tag{4.5}$$

and

$$ds^2\mid_{r=r_{AH}^+}=\frac{4\sin\left(\frac{1}{3}\Psi\right)}{\sin\Psi}\frac{dm}{dv}dv^2+d\Omega_{n+1}^2, \tag{4.6}$$

with $\frac{\pi}{2}<\Psi\left(v\right)<\pi=\arccos\left[-3m\left(v\right)\sqrt{\Lambda}\right]$. Equations (4.5) and (4.6) are exactly the known results [21] for the four dimensional ($n=1$) case.

B. The dynamics of the Horizons

The manner in which the various horizons move can now be inferred. One can rewrite equations (4.1) and (4.3) in the form

$$ds^2 \mid_{r=r^{\pm}_{AH}} \simeq \pm 2L_0 \Gamma^{\pm}_{(AH)n} dv^2 + d\Omega^2_{n+1} \tag{4.7}$$

where

$$\Gamma^{\pm}_{(AH)n} = \frac{k}{2} \alpha_{(\pm)} (p, \varphi) \left[\rho \frac{\sin \frac{1}{3}\varphi}{\sin \varphi} + \sigma \cos \frac{1}{3}\varphi \right]. \tag{4.8}$$

We see then that to $O(L)$ the apparent horizons $r^{\pm}_{AH}$ move with velocities given by

$$\frac{dr^{\pm}_{AH}}{dv} \simeq \pm 2L_0 \Gamma^{\pm}_{(AH)n} \tag{4.9}$$

Similarly the motion of the event horizons $r^{\pm}_{EH}$ can be deduced from equations (3.28) and (3.29). One finds that the velocities of these surfaces are given by

$$\frac{dr^{\pm}_{EH}}{dv} \simeq \pm 2L_0 \Gamma^{\pm}_{(EH)n} \tag{4.10}$$

where the $\Gamma^{\pm}_{(EH)n}$ are obtained by applying to equation (4.8) the transformation $m(v) \to m^*(v) = m(v) - \frac{L_0}{\kappa}$ that turns apparent horizon quantities to event horizon quantities. For the range $\left(\frac{1}{2}\pi < \varphi, \varphi^* < \pi\right)$ considered the quantities $\Gamma^{\pm}_{(AH)n}$ and $\Gamma^{\pm}_{(EH)n}$ are positive. It follows then from (4.9) and (4.10) that for the observer in the region $r^{-}_{AH} < r < r^{+}_{AH}$ both the black hole horizons EH^- and AH^- move with respective velocities $-2L_0\Gamma^{-}_{(EH)n}$ and $-2L_0\Gamma^{-}_{(AH)n}$. For such an observer these velocities are negative Consequently such motion represents in each case a contraction of the respective black hole horizon. Conversely, the same equations show that for the same observer, the cosmological horizons EH^+ and AH^+ are expanding at velocities given by $2L_0\Gamma^{+}_{(EH)n}$ and $2L_0\Gamma^{+}_{(AH)n}$, respectively.

In the event $(n \to 1)$ one recovers the known results [21] for the four dimensional case. And in this limit the results are also consistent with the usual results that as $\Lambda \to 0$, we recover from equations (4.7) and (4.8) the relations

$$\lim_{\substack{n\to 1\\ \Lambda\to 0}} \frac{dr^-_{AH}}{dv} = -2L_0 \tag{4.11}$$

and

$$\lim_{\substack{n\to 1\\ \Lambda\to 0}} \frac{dr^-_{EH}}{dv} = -2L_0 \tag{4.12}$$

V. CONCLUSION

In this discussion we have examined black holes of the Vaidya type in an spatially flat higher dimensional spacetime with a cosmological constant. The analysis revealed the existence of four horizons associated with such a spacetime and identified as the event horizon, EH^- and the apparent horizon AH^- for the black hole, and their cosmological counterparts, EH^+ and AH^+, respectively. We have pointed out, to good order of accuracy, the location of these horizons. Further, from our results, we deduced the structure and discussed the dynamics of these horizons. All our results reduce to already known results under various limits. In particular, it was shown at each stage that for the $n = 1$, our results reduced exactly to those previously obtained [21] for the four dimensional case. It is seen then that the problem of the dynamics of a radiating blackhole in a higher dimensional cosmological background can be sensibly discussed.

An application of our results to the Hawking radiation problem will be the topic of a future discussion.

Acknowledgments:

I would like to thank Fred Adams for some insightful discussions and Gordy Kane, Marty Einhorn and Jean Krisch for their constructive comments.

This work was supported by funds from The University of Michigan.

REFERENCES

[1] A. H. Guth, Phys. Rev. D **23** 347 (1981).

[2] E. Cremmer and B. Julia, Nucl. Phys. B **186**, 412 (1981); A. Salam and J. Strathdee, Ann. Phys. (NY) **141**, 316 (1982).

[3] M. Green, I. Schwarz and E. Witten, *Superstring Theory*, Cambridge Univ. Press, Cambridge, UK, 1986.

[4] D. Sahdev, Phys. Rev. D, **30** 2495 (1984).

[5] Y. B. Zel'dovich and I. D. Novikov, Sov. Aston. **10**, 602 (1967).

[6] S. W. Hawking, Mon. Not. R. Astron, Soc., **152**, 75 (1971).

[7] B. J. Carr and S. W. Hawking, Mon. Not. R. Astron, Soc., **168**, 399 (1974).

[8] B. J. Carr and J. Lindsey, Phys. Rev. D **48**, 543 (1993); **50** 4853 (1994).

[9] D. Gross, M. J. Perry and L. G. Yaffe, Phys. Rev. D **25**, 330 (1982); **36**, 1603 (1987).

[10] F. C. Adams, M. Mbonye and G. Laughlin, Phys. Lett. B **450** 339 (1999)

[11] A. W. Peet, Class. Quantum Grav., **15**, 3291 (1998).

[12] K. Suzuki, Phys. Rev. D **5806** 15 (1998).

[13] A. Chados and S. Detweiler, Gen Relativ. Gravit. **14**, 879 (1982).

[14] G. W. Gibbons and D. L. Wiltshire, Ann. Phys. **167**, 201 (1986).

[15] R. C. Myers and M. J. Perry, Ann. Phys. (NY) **172**, 304 (1986).

[16] P. O. Mazur, J. Math. Phys., **28**, 406 (1987).

[17] B. R. Iyer and C. V. Vishveshwara, Pramana J. Phys., **32**, 749 (1989).

[18] R. L. Mallett, Phys. Rev. D **31**, 416 (1985).

[19] L. K. Patel and L S. Desai, Pramana J. Phys. **48**, 819 (1997).

[20] Zhen-Qiang Tan and You-Gen Shen, Il Nuovo Cimento **113 B**, 339 (1998).

[21] R. L. Mallett, Phys. Rev D **33**, 2201 (1986).

[22] B. Carter, G*eneral Relativity*, (edited by S. W. Hawking and I. Isreal) Cambridge Univ. Press, Cambrige, UK (1979).

[23] R. Capovilla and J. Guven, Phys. Rev. D **51**, 6736 (1995).

[24] E. Zafiris, Phys. Rev. D **58**, 043509-1 (1998).

[25] A. Raychaudhuri, Phys. Rev. **98**, 1123 (1955).

[26] J. W. York, Jr., in Quantum Theory of Gravity: Essays in Honor of the Sixtieth Birthday of Bryce S. DeWitt, edited by S. Christensen (Hilger, Bristol, 1984).